Math 4 Honors
Unit 7 Test Review

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Date ______

1. In the bottom of an hourglass a conical pile of sand is formed at the rate of 12 cubic inches per minute. The radius of the base of the pile is always equal to 1/2 its altitude. How fast is the altitude rising when it is 6 inches deep?

$\frac{dV}{dt} = 12\ in^3/min$

$r = \frac{1}{2}h$

$\frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(\frac{1}{2}h)^2 \cdot h = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \cdot \frac{dh}{dt}$

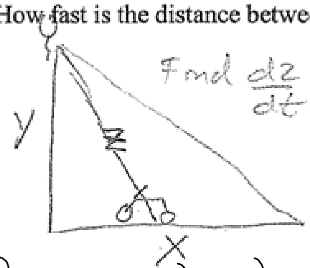
$12 = \frac{1}{4}\pi \cdot 6^2 \cdot \frac{dh}{dt}$

$12 = 9\pi \cdot \frac{dh}{dt}$

$\frac{12}{9\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{4}{3\pi}\ in/min}$

2. A balloon is rising vertically above a level, straight road at a constant rate of 1 ft./sec. Just when the balloon is 65 feet above the ground, a bicycle moving at a constant rate of 17 ft./sec. passes under it. How fast is the distance between the bicycle and balloon increasing 3 seconds later?



Find $\frac{dz}{dt}$

$\frac{dy}{dt} = 1\ ft/sec$

$51^2 + (65+3)^2 = z^2$

$x^2 + y^2 = z^2$

$\cancel{2}x \cdot \frac{dx}{dt} + \cancel{2}y \cdot \frac{dy}{dt} = \cancel{2}z \cdot \frac{dz}{dt}$

$51 \cdot 17 + 68 \cdot 1 = 85 \cdot \frac{dz}{dt}$

$935 = 85 \cdot \frac{dz}{dt}$

$\boxed{11\ ft/sec = \frac{dz}{dt}}$

3. For each of the following find $\frac{dy}{dx}$.

a. $x^{\frac{1}{2}} - y^{\frac{1}{2}} = 1$

$\frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}y^{-\frac{1}{2}} \cdot \frac{dy}{dx} = 0$

$-\frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$

$\frac{dy}{dx} = \sqrt{\frac{y}{x}}$

d. $y = x\sqrt{x^2+1} = x(x^2+1)^{\frac{1}{2}}$

$\frac{dy}{dx} = 1(x^2+1)^{\frac{1}{2}} + \frac{1}{2}x(x^2+1)^{-\frac{1}{2}} \cdot 2x$

$\frac{dy}{dx} = (x^2+1)^{\frac{1}{2}} + x^2(x^2+1)^{-\frac{1}{2}}$

$\frac{dy}{dx} = (x^2+1)^{-\frac{1}{2}}[(x^2+1)^1 + x^2]$

$\frac{dy}{dx} = (x^2+1)^{-\frac{1}{2}}(2x^2+1)$

b. $x^3 - xy + y^3 = 1$

$3x^2 - (x\frac{dy}{dx} + y \cdot 1) + 3y^2 \cdot \frac{dy}{dx} = 0$

$3x^2 - x\frac{dy}{dx} - y + 3y^2 \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx}(3y^2 - x) = y - 3x^2$

$\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$

e. $y = \ln(3x+2)$

$\frac{dy}{dx} = \frac{1}{3x+2} \cdot 3$

$\frac{dy}{dx} = \frac{3}{3x+2}$

OVER →

c. $y = \sin^3(2x+3)\tan(5x+2)$

$\frac{dy}{dx} = \sin^3(2x+3)\sec^2(5x+2)\cdot 5 + \tan(5x+2)\cdot 3\sin^2(2x+3)\cdot\cos(2x+3)\cdot 2$

$= \sin^2(2x+3)[5\sin(2x+3)\sec^2(5x+2) + 6\tan(5x+2)\cos(2x+3)]$

f. $y = 10u^2 - 3u + 8$
$u = 4x - 5$

$\frac{dy}{du} = 20u - 3$

$\frac{du}{dx} = 4$

$\frac{dy}{dx} = 4(20u-3)$
$= 80u - 12$

$\frac{dy}{dx} = 80(4x-5) - 12$

$\frac{dy}{dx} = 320x - 400 - 12$

$\frac{dy}{dx} = 320x - 412$

4. Write the equation of the tangent line at the point (1, 0) for the equation in #3b.

$\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$

$m_{T.L.} = \frac{0 - 3\cdot 1^2}{3\cdot 0^2 - 1}$

$= \frac{-3}{-1}$

$= 3$

$y = 3x + b$

$0 = 3\cdot 1 + b$

$b = -3$

$y = 3x - 3$

5. $y = \dfrac{x^2}{x+2}$

a. Find the x-intercept.
b. Find the vertical asymptote.
c. Find $f'(x)$.
d. What are the maximum and minimum points (if any)?

a. $x^2 = 0$
$x = 0$
$(0, 0)$

b. $x = -2$

x	$-$	$-$	$-$	$+$
$x+4$	$-$	$+$	$+$	$+$
$(x+2)^2$	$+$	$+$	$+$	$+$
	$+$	-4 $-$	-2 $-$	0 $+$

MAX MIN

c. $y' = \frac{2x(x+2) - 1\cdot x^2}{(x+2)^2}$

$= \frac{2x^2 + 4x - x^2}{(x+2)^2}$

$= \frac{x^2 + 4x}{(x+2)^2}$

$= \frac{x(x+4)}{(x+2)^2}$

d. $x(x+4) = 0$

$x = 0$, $x = -4$

$(0, 0)$ MIN $(-4, -8)$ MAX